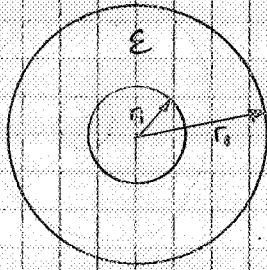


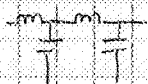
Circuit Cavity QEDTransmission line

$$\vec{E}(s, \varphi, z) = \frac{V_0}{\ln(r_o/r_i)} \frac{\vec{e}_s}{s} e^{i\omega t - i\beta z}$$

$$\vec{H}(s, \varphi, z) = \frac{I_0}{2\pi} \frac{\vec{e}_\varphi}{s} e^{i\omega t - i\beta z} \quad \text{TEM modes}$$

$$Z_0 = \frac{V_0}{I_0} = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln(r_o/r_i)}{2\pi} \quad \beta = \frac{\omega}{\sqrt{\epsilon\mu}} \quad \sqrt{\frac{\mu}{\epsilon_0}} = 377 \Omega$$

no dispersion



capacitance per unit length

$$C^* = \frac{1}{V_0^2} \int_{\Sigma} \epsilon_r \epsilon_0 \vec{E} \cdot \vec{E}^* dx dy = \frac{2\pi}{\ln(r_o/r_i)} \epsilon_r \epsilon_0$$

inductance per unit length

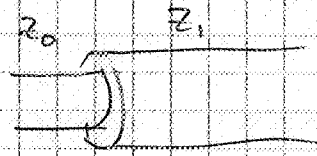
$$L^* = \frac{1}{I_0^2} \int_{\Sigma} \mu_r \mu_0 \vec{H} \cdot \vec{H}^* dx dy = \frac{\ln(r_o/r_i)}{2\pi} \mu_r \mu_0$$

$$Z_0 = \sqrt{\frac{L^*}{C^*}}$$

semi infinite transmission line is physically identical to a resistor of $R = Z_0$

LII, 2

impedance mismatch

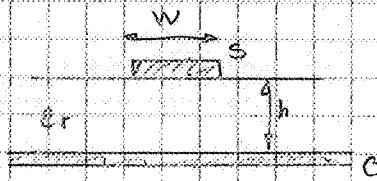


reflection coefficient

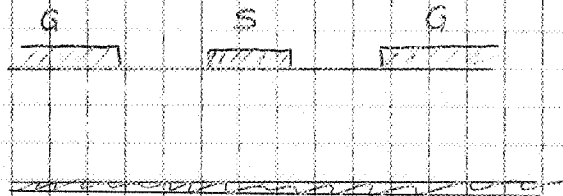
$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

other transmission lines

microstrip



coplanar waveguide



Transmission line resonator

slides

general resonance

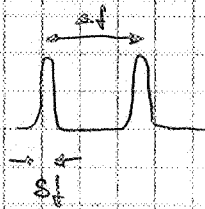


$$S = t e^{i\beta L} t + t e^{i\beta L} r e^{i\beta L} r e^{i\beta L} t + \dots$$

$$= t^2 e^{i\beta L} \left(\sum_n (r e^{i\beta L})^{2n} \right)$$

$$= \frac{1}{1 - (r e^{i\beta L})^2}$$

geometrical series

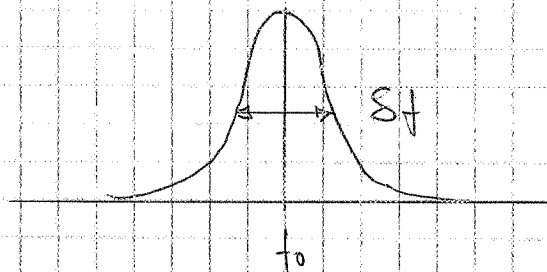


resonance if $\beta L \approx m\pi$

$$S_L = \frac{1}{f - f_0 - i\gamma}$$

$$|S_L| = \frac{1}{(f - f_0)^2 + \gamma^2}$$

Lorentz curve



δf full width halfmax

$$\delta f = 2\gamma$$

Quality factor $Q = \frac{f_0}{\delta f}$

$$\text{Finesse} = \frac{\pi \delta f}{\delta f}$$

L11.4

transmission line cavity

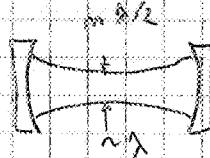
slide

$$Q_{\text{ext}} = \frac{\pi}{4m} \frac{1}{(C_{\text{ext}} 2\pi f_0 Z_0)^2}$$

$$f_0 = \frac{c/\sqrt{\epsilon_0 \mu_0}}{2L}$$

$$Q_{\text{tot}}^{-1} = Q_{\text{ext}}^{-1} + Q_{\text{int}}^{-1}$$

optical cavity



fields for one photon

$$\frac{1}{2} h f_0 = \frac{\epsilon_0}{2} \int \vec{E}^2 dV \approx \frac{\epsilon_0}{2} E_0 V$$

$$E_0 = \sqrt{\frac{h f_0}{V \epsilon_0}}$$

$$\text{cavity volume } V = \pi r^2 \lambda / 2$$

$$\lambda = \frac{c}{f_0}$$

$$\Rightarrow E_0 = \frac{f_0}{r} \sqrt{\frac{h}{\frac{\pi}{2} \epsilon_0 c}}$$

$$f_0 = 5 \text{ GHz} \quad r = 10 \mu\text{m}$$

$$h f_0 = 3.3 \cdot 10^{-24} \text{ J}$$

$$E_0 = 0.2 \text{ V/m}$$

L11,5

comparing to conventional cavity

$$V \propto \lambda^2$$

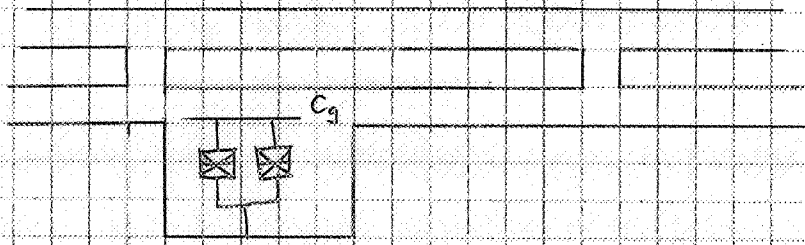
E_0 is larger by
factor $\frac{\lambda}{r}$

superconductivity $Q \sim 5 \cdot 10^5$

slide

$$5 \text{ GHz} \cdot h = k_B \cdot 240 \text{ mK}$$

L11.6

circuit QED

Cooper-pair box Hamiltonian

$$H_q = -\frac{1}{2} E_{c1} \sigma_z - \frac{1}{2} E_J \sigma_x$$

$$\text{with } E_{c1} = 4E_c (1 - 2n_g) \quad n_g = \frac{C_g V_g}{2e}$$

$$V_g = \frac{C_g}{C_L} (V_g^{DC} + V^{AC})$$

$$V^{AC} = V^0 (a + a^\dagger) \quad V^0 = E^0 r$$

$$H_q = -2E_c (1 - 2n_g^{DC}) \sigma_z - \frac{E_J}{2} \sigma_x \\ - e \frac{C_g}{C_L} V^0 (a + a^\dagger) (1 - 2n_g^{DC} - \sigma_z)$$

In the qubit eigenbasis

$$H_{\text{qubit}} = \hbar \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar \Omega}{2} \sigma_z \\ - e \frac{C_g}{C_L} V^0 (a + a^\dagger) \left(1 - 2n_g^{DC} - \cos(\theta) \sigma_z + \sin(\theta) \sigma_x \right)$$

$$\Omega = \sqrt{E_J^2 + E_{c1}^2} \quad \theta = \arctan \left(\frac{E_J}{E_{c1}} \right)$$

L11.7

for $n_g = \frac{1}{2}$ and neglecting fast rotating terms

$$H = \hbar \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar \Omega}{2} \sigma_z + \hbar g \left(a^\dagger \sigma_- + a \sigma_+ \right)$$

Jaynes-Cummings Hamiltonian

coupling strength $g = \frac{e}{\hbar} \frac{C_g}{C_z} V_0$

note, the term $a^\dagger \sigma_- + a \sigma_+$ only couples the term $|g, n\rangle$ and $|e, n-1\rangle$ i.e terms with n excitations

$$|+, n\rangle = \cos \theta_{jc} |e, n-1\rangle + \sin \theta_{jc} |g, n\rangle$$

$$|-, n\rangle = \sin \theta_{jc} |e, n-1\rangle - \cos \theta_{jc} |g, n\rangle$$

where θ_{jc} is the mixing angle

$$\tan(2\theta_{jc}) = \frac{2g\sqrt{n}}{\Delta} \quad \Delta = \Omega - \omega_r$$

Eigenenergies

$$E_{\pm, n} = n\hbar\omega_r \pm \frac{\hbar}{2} \sqrt{4g^2 n + \Delta^2}$$